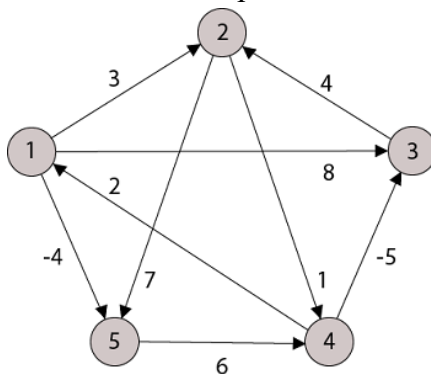


長庚大學 110 學年度第二學期資工所博士班演算法資格考

1. Please write down your student ID and name on the answer sheet.
 2. Please indicate the number of each your answer that is relative to the problem.
 3. Any form of cheating will lead to fail.
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Please select five problems to answer. Total score of this exam is 100. Maximum deduction of 20 points for each problem that your answer.

1. If $|V|$ is the number of vertices in the directed graph, and $|E|$ is the number of edges, what is the running time of Dijkstra's algorithm in $O(\)$ notation? Give a brief justification for your answer. You should also briefly describe any assumptions you are making about the implementation that would affect the answer.
2. Solve the all-pairs shortest path problem on the following weighted, directed graph using Floyd-Warshall algorithm. Please show the matrices of distance D and predecessor matrices Π in each iteration of the loop.



3. Given a positive integer n , we consider the following problem: Making change for n cents using the fewest number of coins. Assume that the value of each coin is a positive integer. Describe a greedy algorithm to make change consisting of quarters (25 cents), dimes (10 cents), nickels (5 cents) and pennies (1 cent). Prove that your algorithm yields an optimal solution to the given make change problem.
4. In the longest-common-subsequence problem, we are given two sequences $X = \langle x_1, x_2, \dots, x_m \rangle$ and $Y = \langle y_1, y_2, \dots, y_n \rangle$ and try to find a maximum-length common subsequence of X and Y . The LCS problem can be solved efficiently using dynamic programming. Please show the optimal substructure of an LCS and prove its correctness.
5. Show that Quicksort's best-case and Mergesort's worst-case running time is $\Omega(n \log n)$
6. Let f be a flow in flow network G with source s and sink t , and let (S, T) be any cut of G . Then the net flow across (S, T) is $f(S, T) = |f|$. Please prove its correctness.