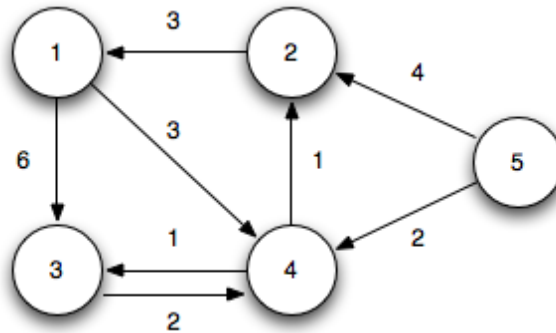


長庚大學 109 學年度第一學期資工所博士班演算法資格考

1. Please write down your student ID and name on the answer sheet.
2. Please indicate the number of each your answer that is relative to the problem.
3. Any form of cheating will lead to fail.

Please select five problems to answer. Total score of this exam is 100. Maximum deduction of 20 points for each problem that your answer.

1. Given a positive integer n , we consider the following problem: Making change for n cents using the fewest number of coins. Assume that the value of each coin is a positive integer. Describe a greedy algorithm to make change consisting of quarters (25 cents), dimes (10 cents), nickels (5 cents) and pennies (1 cent). Prove that your algorithm yields an optimal solution to the given make change problem.
2. Solve the all-pairs shortest path problem on the following weighted, directed graph using Floyd-Warshall algorithm. Please show the matrices of distance D and predecessor matrices Π in each iteration of the loop.



3. Consider a modification of the rod-cutting problem in which, in addition to a price p_i for each rod, each cut incurs a fixed cost of c . The revenue associated with a solution is now the sum of prices of the pieces minus the cost of making the cut.
 - (a) Give a dynamic-programming algorithm to solve this modified problem, including the mathematical expression for the maximum revenue and the pseudocode
 - (b) Express your answer as an array of maximum revenue $R[1.. 10]$ with $c=1$

Length i	1	2	3	4	5	6	7	8	9	10
Price p_i	2	5	6	8	11	12	15	18	19	20

4. Let f be a flow in flow network G with source s and sink t , and let (S, T) be any cut of G . Then the net flow across (S, T) is $f(S, T) = |f|$. Please proof the description above.
5. Please explain the time complexity of Bellman-ford algorithm, DFS algorithm and BFS algorithm *in detail* (especially the differences between the adjacency matrix and list used to present the graph)
6. A file contains the following characters with the frequencies as shown. (Characters, Frequency) = {(a, 10), (e, 15), (i, 12), (o, 3), (u, 4), (s, 13), (t, 1)}. If Huffman Coding is used for data compression. Please determine the following results.
 - (a) Huffman Code for each character
 - (b) Average code length
 - (c) Length of Huffman encoded message (in bits)