長庚大學 110 學年度第一學期資工所博士班演算法資格考

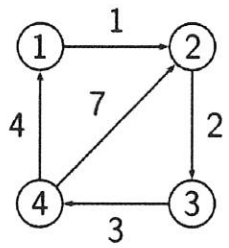
1. Please write down your student ID and name on the answer sheet.
2. Please indicate the number of each your answer that is relative to the problem.
3. Any form of cheating will lead to fail.

Please select five problems to answer. Total score of this exam is 100. Maximum deduction of 20 points for each problem that your answer.

1. Consider a modification of rod-cutting problem in which, in addition to a price pi for each rod, each cut incurs a fixed cost of c. The revenue associated with a solution is now the sum of the prices of the pieces minus the cost of making the cut.
   (a) Give a dynamic programming algorithm to solve the modified problem, including the mathematical expression for the maximum revenue and the pseudocode.
   (b) Express your answer as an array of maximum revenue R[1..10] with c=0.3
   (c) Describe a series of cuts that given the maximum revenue for i=8, 9 and 10 inch rod, respectively.

Length i	1	2	3	4	5	6	7	8	9	10
Price pi	0.5	1.5	1.8	2.5	3.1	3.5	4.2	4.5	4.8	5.2

2. Solve the all-pairs shortest path problem on the following weighted, directed graph using Floyd-Warshall algorithm. Please show the matrices of distance D and predecessor matrices Π in each iteration of the loop.



3. (A) Please list the worst and average ***time*** complexity of Quicksort sort, Merge sort, and Heap sort, respectively. (B) Please describe and explain the ***space*** complexity of Quicksort sort, Merge sort and Heap sort, respectively.

4. What is the time complexity of Greedy based solution of Activity Selection Problem? Please explain your answer in detail.

5. You are given a directed acyclic graph $G = (V, E)$. Each edge $(u, v) \in E$ of this graph is associated with a weight $w(u, v)$, which can be any positive real number. The cost of a path is defined to be the sum of the weights of the edges along the path. Present an efficient algorithm to find the path with the maximum cost in G. Justify your algorithm's correctness and establish its running time.

6. Consider the network flow problem with the following edge capacities, c(u,v) for edge (u,v): c(s,1)=16, c(s,2)=13, c(1,3)=12, c(3,t)=20, c(2,1)=4, c(3,2)=9, c(2,4)=14, c(4,3)=7, c(4,t)=4
   (a) Draw the network.
   (b) Run the Ford-Fulkerson algorithm to find the maximum flow. Show each residual graph.
   (c) Show the minimum cut.