長庚大學 112 學年度第一學期資工所博士班演算法資格考

1. Please write down your student ID and name on the answer sheet.
2. Please indicate the number of each your answer that is relative to the problem.
3. Any form of cheating will lead to fail.

Please select five problems to answer. Total score of this exam is 100. Maximum deduction of 20 points for each problem that your answer.

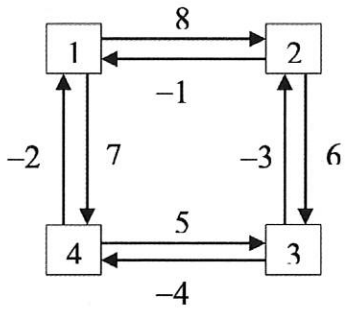
1. Consider the problem of making change for n cents using the fewest number of coins. Assume that each coin's value is an integer.
   a. Describe a greedy algorithm to make change consisting of quarters (25 cents), dimes (10 cents), nickels (5 cents), and pennies (1 cents). Prove that your algorithm yields an optimal solution.
   b. Give a set of coin denominations for which the greedy algorithm does not yield an optimal solution. Your set should include a penny so that there is a solution for every value of n.

2. Please explain the time complexity of Bellman-ford algorithm and breadth first search (BFS) algorithm ***in detail*** (especially the differences between the *adjacency matrix* and *list* used to present the graph)

3. (A) Two of the most common divide-and-conquer sorting algorithms are quicksort and mergesort. In practice quicksort is often used for sorting data in main storage rather than mergesort. Give a reason why quicksort is likely to be the preferred sorting algorithm for this application.; (B) Quicksort's worst-case running time is $O(n^2)$, but it has an expected running time of O(nlogn) if the partition function works well. What needs to be true about the partition function in order for the running time to be O(nlogn)? In practice, how can we ensure that this happens?

4. A file contains the following characters with the frequencies as shown. (Characters, Frequency)={(a, 20), (e, 15), (i, 12), (o, 3), (u, 5), (s, 10), (t, 1)}. If Huffman Coding is used for data compression. Please determine the following results.
   a. Huffman Code for each character
   b. Average code length
   c. Length of Huffman encoded message (in bits)

5. Solve the all-pairs shortest path problem on the following weighted, directed graph using Floyd-Warshall algorithm. Please show the matrices of distance D and predecessor matrices Π in each iteration of the loop.



6. Consider the network flow problem with the following edge capacities, c(u,v) for edge (u,v): $c(s,v_1)=16$, $c(s,v_2)=13$, $c(v_1,v_2)=10$, $c(v_2,v_1)=4$, $c(v_1,v_3)=12$, $c(v_2,v_4)=14$, $c(v_3,v_2)=9$, $c(v_4,v_3)=7$, $c(v_3,t)=20$, $c(v_4,t)=4$
   (a) Draw the network.
   (b) Run the Ford-Fulkerson algorithm to find the maximum flow. Show each residual graph after each augmentation.
   (c) Show the minimum cut.