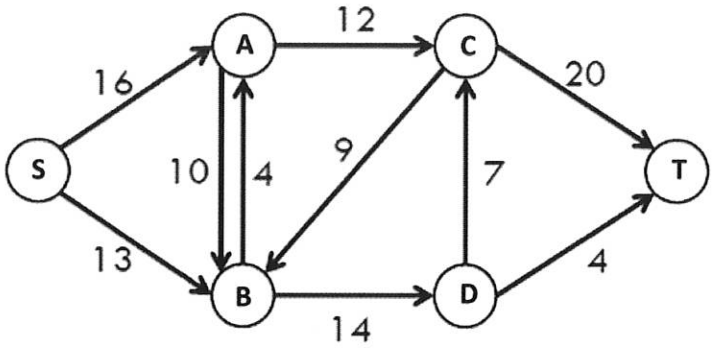
長庚大學 109 學年度第二學期資工所博士班演算法資格考

1. Please write down your student ID and name on the answer sheet.
2. Please indicate the number of each your answer that is relative to the problem.
3. Any form of cheating will lead to fail.

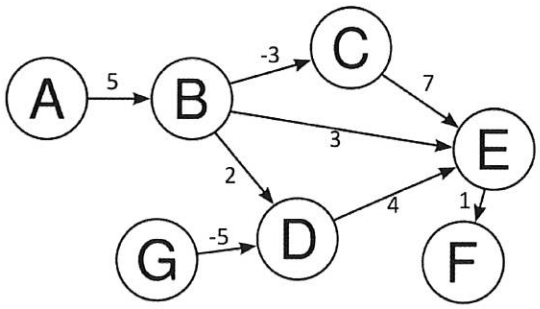
Please select five problems to answer. Total score of this exam is 100. Maximum deduction of 20 points for each problem that your answer.

1. Given a graph which represents a flow network where every edge has a capacity. Please apply the Ford-Fulkerson algorithm (use the shortest augmenting path method to choose the augmenting path in each iteration) to find a maximum S − T flow.



2. To prove that the greedy algorithm Huffman is correct, we show that the problem of determining an optimal prefix code exhibits the greedy choice and optimal substructure properties. **Lemma:** Let C be an alphabet in which each character c ∈ C has frequency c.freq. Let x and y be two characters in C having the lowest frequencies. Then there exists an optimal prefix code for C in which the codewords for x and y have the same length and differ only in the last bit.

3. (A) Please explain the time complexity of Bellman-ford algorithm, Dijkstra's algorithm for calculating the single source shortest path. (B) What is the best approach to obtain the SSSP from source of vertex B in the following graph? Please describe the reason and use the best approach to find the SSSP from source as B.



4. (A) Please prove that the worst-case behavior of Quicksort is $O(n^2)$ and the average computing time of Quicksort is O(nlogn). (B) Sorting the list [1, 12, 5, 26, 7, 14, 3, 7, 2] using the QuickSort algorithm. The final result should be in ascending order. Each step should be described in detail.

5. (A) Please list the worst and average ***time*** complexity of Insertion sort, Merge sort, and Heap sort, respectively. (B) Please describe and explain the ***space*** complexity of Insertion sort, Merge sort and Quick sort, Heap sort, respectively.

6. Trace the dynamic programming algorithm for the longest common subsequence problem with strings X=GCCCTAGCG and Y = GCGCAATG. Complete all the entries in the table below, and also build ***all*** of the optimal solutions.