長庚大學 108 學年度第二學期資工所博士班演算法資格考

1. Please write down your student ID and name on the answer sheet.
2. Please indicate the number of each your answer that is relative to the problem.
3. Any form of cheating will lead to fail.

Please select five problems to answer. Total score of this exam is 100. Maximum deduction of 20 points for each problem that your answer.

1. Consider the *Maximum alternating Sum Subsequence (MASS)* problem. Given a sequence $S=[x_1, x_2, …, x_n]$ of positive integers, find the subsequence $A=[x_{i1}, x_{i2},…, x_{ik}]$, where $i_1< i_2 <…< i_k$, that maximizes the alternating sum

$$x_{i1} - x_{i2} + x_{i3} - x_{i4+}…\pm x_{ik}$$

   For example, if S= [4, 9, 2, 4, 1, 3, 7], the MASS is A= [9, 2, 4, 1, 7] which evaluates to 9-2+4-1+7=17. If S=[7, 6, 5, 4, 3, 2, 1], the MASS is A=[7]. Clearly, the length of the MASS is depend on the actual of values in S. Assume that the length of S is at least one, and all its elements are integers greater than zero. Please use the dynamic programming algorithm to solve this problem. (Hint: it helps to treat subsequence for even and odd lengths separately.)

2. Solve the all-pairs shortest path problem on the following weighted, directed graph using Floyd-Warshall algorithm.
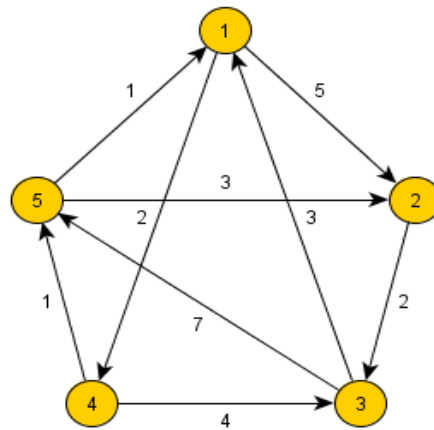
3. In this question, consider only 4 sorting algorithms: Insertion Sort, Quick Sort, Merge Sort, and Radix Sort. Choose the fastest sorting method that is suitable for each scenario and explain the reason.
   (a) You are compiling a list of students (ID, GPA) for a corporate visiting activity. However, you are facing a problem in the amount of memory available for your old computer. After loading all students in memory, the extra memory available can only hold up to 25% of the total students. Which sorting method should be used to sort all students based on GPA (no fixed precision)?
   (b) After your success in creating the list for the activity, you are hired as a part-time assistant in CGU to manage a student database. There are student records, already sorted by name. However, we want a list of students first ordered by age. For all students with the same age, we want them to be ordered by name. In other words, we need to preserve the ordering by name as we sort the data by age.
   (c) After finishing the part-time job in CGU, you are invited to be a teaching assistant. You have just finished marking the final exam papers randomly. You want to determine your students' grades, so you need to sort the students in order of marks. The marks have no fixed precision.

4. What is the time complexity of Greedy based solution of Activity Selection Problem? Please explain your answer in detail.

5. Please describe what is Bellman-Ford algorithm for the single source shortest path problem? If the input graph G contains a negative cycle, is bellman ford method still working? Please explain it in detail.

6. Consider the network flow problem with the following edge capacities, c(u,v) for edge (u,v): c(s,2)=2, c(s,3)=13, c(2,5)=12, c(2,4)=10, c(3,4)=5, c(3,7)=6, c(4,5)=1, c(4,6)=1, c(6,5)=2, c(6,7)=3, c(5,t)=6, c(7,t)=2
   (a) Draw the network.
   (b) Run the Ford-Fulkerson algorithm to find the maximum flow. Show each residual graph.
   (c) Show the minimum cut.