長庚大學 105 學年度第二學期電機所博士班演算法資格考

1. Please write down your student ID and name on the answer sheet.
2. Please indicate the number of each your answer that is relative to the problem.
3. Any form of cheating will lead to fail.

Please select five problems to answer. Total score of this exam is 100. Maximum deduction of 20 points for each problem that your answer.

1. Suppose a data file contains a sequence of 8-bit characters such that all 256 characters are about as common: the maximum character frequency is less than twice the minimum character frequency. Prove that Huffman coding in this case also produces 8-bit code for each character.

2. (a) Please proof that $n_0 = n_2 + 1$ in a binary tree for n_0 is the number of leave nodes, n_1 is the number of nodes with one child and n_2 is the number of nodes with 2 children. (b) If a complete binary tree with n nodes is represented sequentially in an array, then for any node with index i, $1 \leq i \leq n$. If we use the same scenario for a skewed tree of depth k, how many spaces in the array will be wasted?

3. Please describe what is Bellman-Ford algorithm for the single source shortest path problem? If the input graph G contains a negative cycle, is bellman ford method still working? Please explain it in detail.

4. You are given a directed acyclic graph $G = (V, E)$. Each edge $(u, v) \in E$ of this graph is associated with a weight $w(u, v)$, which can be any positive real number. The cost of a path is defined to be the sum of the weights of the edges along the path. Present an efficient algorithm that computes the maximum cost of any path in G. Justify your algorithm's correctness and establish its running time.

5. You are driving from Taoyuan to Kaohsiung, using one or more rental cars for the trip. Along the way, you will visit cities $c_1 \ldots c_n$ in order (here, c_1 is Taoyuan and c_n is Kaohsiung). You have a fee schedule $f(i, j)$ that gives the cost of a rental that is picked up in city ci and dropped off in city c_j, $j > i$. Note that these costs are arbitrary and possibly non-monotonic; for example, it may cost NTD\$1000 for a rental from c_1 to c_2 but only \$500 for a rental from c_1 to c_3. Give an algorithm to choose a set of rentals that minimize the total cost of your trip. Note that you can rent only one car at a time, and that you can never be without a car. Hence, a solution is a set of non-overlapping rentals that span all cities.

6. Prove that the following greedy algorithm is optimal, or give a counterexample to show that it is not. Also, analyze the worst-case running time of the algorithm.

   Problem: You are given a set of n jobs, each of which runs in unit time. Job i has an integer-valued deadline time $d_i \geq 0$ and a real-valued bonus $b_i \geq 0$. Jobs may be scheduled to start at any integer time (0, 1, 2, etc), and only one job may run at a time. If job i completes at or before time d_i, then it obtains bonus b_i; otherwise, it obtains no bonus. The goal is to schedule all jobs so as to maximize the total bonus obtained.

   Algorithm: Define slot k in the schedule to run from time $k - 1$ to time k. First, sort the jobs so that $b_1 \geq b_2 \geq \cdots \geq b_n$. Then, add each job to the schedule in turn. When adding job i, if any time slot between 0 and d_i is available, then schedule i in the latest such slot. Otherwise, schedule job i in the latest available slot $\leq n$.