長庚大學 103 學年度第二學期電機所博士班演算法資格考

1. Please write down your student ID and name on the answer sheet.
2. Please indicate the number of each your answer that is relative to the problem.
3. Any form of cheating will lead to fail.

Please select five problems to answer. Total score of this exam is 100. Maximum deduction of 20 points for each problem that your answer.

1. (a) Please proof that $n_0=n_2+1$ in a binary tree for n_0 is the number of leave nodes, n_1 is the number of nodes with one child and n_2 is the number of nodes with 2 children. (b) If a complete binary tree with n nodes is represented sequentially in an array, then for any node with index i, $1 \le i \le n$. If we use the same scenario for a skewed tree of depth k, how many spaces in the array will be wasted?
2. Consider a Dynamic Programming (DP) algorithm which takes as input an array A of n elements. Subproblems of the given problem are contiguous subarrays [Ai , . . . , Aj] where $1 \le i \le j \le n$. The DP algorithm computes solutions to all possible subproblems before it returns the desired result for the given problem [A1, . . . , An]. The number of subproblems for which solutions are computed by the DP algorithm is then:
   (a) $\Theta(\log n)$
   (b) $\Theta(n \log n)$
   (c) $\Theta(n^2)$
   (d) $\Theta(n^3)$
   Which one is the correct answer? Please explain your reason.
3. Let G = (V, E) be a connected, undirected graph with edge-weight function w : $E \rightarrow R$, and assume all edge weights are distinct. Consider a cycle $(v_1, v_2, \ldots, v_k, v_{k+1})$ in G, where $v_{k+1} = v_1$, and let (v_i, v_{i+1}) be the edge in the cycle with the largest edge weight. Prove that (v_i, v_{i+1}) does not belong to the minimum spanning tree T of G.
4. If |V| is the number of vertices in the directed graph, and |E| is the number of edges, what is the running time of Dijkstra's algorithm in O() notation? Give a brief justification for your answer. You should also briefly describe any assumptions you are making about the implementation that would affect the answer.
5. (a) Two of the most common divide-and-conquer sorting algorithms are quicksort and mergesort. In practice quicksort is often used for sorting data in main storage rather than mergesort. Give a reason why quicksort is likely to be the preferred sorting algorithm for this application.; (b) Quicksort's worst-case running time is $O(n^2)$, but it has an expected running time of O(nlogn) if the partition function works well. What needs to be true about the partition function in order for the running time to be O(nlogn)? In practice, how can we ensure that this happens?
6. Give the matrices obtained while applying Floyd-Warshall on the below graph step by step.