長庚大學 103 學年度第一學期電機所博士班演算法資格考

1. Please write down your student ID and name on the answer sheet.
2. Please indicate the number of each your answer that is relative to the problem.
3. Any form of cheating will lead to fail.

Please select five problems to answer. Total score of this exam is 100. Maximum deduction of 20 points for each problem that your answer.

1. Let $G=(V, E)$ be a connected undirected graph with edge-weight function $w: E \rightarrow R$. Let w_{min} and w_{max} denote the minimum and maximum weights of the edge in the graph respectively. Do not assume that the edge weights in G are distinct or nonnegative. The following statements may or may not be correct. In each case, either prove the statement is correct or give a counterexample if it is incorrect.
   (a) If the graph G has more than |V|-1 edges and there is a unique edge having the largest weight w_{max}, then this edge cannot be part of any minimum spanning tree.
   (b) Any edge e with weight w_{min}, must be part of some MST.
   (c) If G has a cycle and there is unique edge e which has the minimum weight on this cycle, then e must be part of every MST.
   (d) Suppose the edge weights are nonnegative. Then the shortest path between two vertices must be part of some MST.
2. Show that quicksort's best-case running time is $\Omega(n\lg n)$ and the worst case is $\Omega(n^2)$.
3. Let f be a flow in flow network G with source s and sink t, and let (S, T) be any cut of G. Then the net flow across (S, T) is $f(S, T)=|f|$. Please proof the description above.
4. Please use Huffman code to encode a text with the characters with the frequencies: a(37), b(18), c(29), d(13), e(30), f(17), g(6). And how many bits will you use to present the message? In comparison with fixed-length code, how many percentages of storage will you save?
5. In the traveling-salesman problem (TSP), a salesman must visit n cities. Modeling the problem as a complete graph on n vertices, we can say that the salesman wishes to make a tour or a hamiltonian cycle, visiting each city exactly only once and finishing at the city he starts from. The salesman incurs a nonnegative integer cost $c(i, j)$ to travel from city i to city j, and the salesman wishes to make a tour whose total cost is minimum, where the total cost k is the sum of the individual costs along the edges of the tour.
   (a) Please prove TSP $\in$ NP
   (b) A hamiltonian cycle in a graph is a cycle that visits every vertex exactly once. Define the language HAM-CYCLE={<G>: there is a hamiltonian cycle in G}. Assuming that HAM-CYCLE is complete for the class NP, prove that TSP is NP-complete.
6. Give asymptotic upper and lower bounds for T(n) in each of the following recurrences. Assume that T(n) is constant for n<=2. Make your bound as tight as possible, and justify your answers.
   (a) $T(n)=2T(n/4)+\sqrt{n}$
   (b) $T(n)=T(9n/10)+n$
   (c) $T(n)=7T(n/3)+n^2$
   (d) $T(n)=T(\sqrt{n})+1$