長庚大學 101 學年度第二學期電機所博士班演算法資格考

1. 請於答案卷第一頁依序寫下學號、姓名。
2. 請詳細閱讀下列試題，並請標明題號依試題順序將答案書寫於答案卷上。
3. 任何形式的作弊，本資格考以 Fail 論。

請選擇五題作答。本次考試總分為 100 分，每錯一題至多扣 20 分，扣至 0 分為止。

1. Let G be an arbitrary weighted, directed graph. Please describe an algorithm to detect if G existed negative cycles. And explain the time complexity of your method.

2. Please describe any possible modification of insertion sort to let its worst case is O(nlogn), please proof it.

3. Show that if a node in a binary search tree has two children, then its successor has no left child and its predecessor has no right child. (hint: Successor means the next node according to in-order traversal)

4. Let G=(V,E) be a connected, undirected graph with a real valued weight function w defined on E. Let A be a subset of E that is included in some MST for G, let (S, V-S) be any cut of G that respects A, and let (u,v) be a light edge crossing (S, V-S). Then, edge (u,v) is safe for A. Please prove the correctness.

5. For a sequence of operations to be performed on a data structure, the i_{th} operation costs i when i is an exact power of 2, costs 1 otherwise. Show that the cost after n operations is O(n). (Amortized Analysis)

6. We know that finding a smallest vertex cover is an NP-complete problem. Can you show that, in a given graph G=(V,E), find out a maximum subset S of E such that any two nodes in S are not adjacent in G, is a NP-Complete problem.

7. Please give a recursive algorithm MATRIX-CHAIN-MULTIPLY(*A, s, i, j*) that actually performs the optimal matrix-chain multiplication, given the sequence of matrices <A1, A2, A3,…., An>, the s table computed for internal use, and the indices i and j. (The initial call would be MATRIX-CHAIN-MULTIPLY(*A, s, 1, n*).)