長庚大學 101 學年度第一學期電機所博士班演算法資格考

1. 請於答案卷第一頁依序寫下學號、姓名。
2. 請詳細閱讀下列試題，並請標明題號依試題順序將答案書寫於答案卷上。
3. 任何形式的作弊，本資格考以 Fail 論。

請選擇五題作答。本次考試總分為 100 分，每錯一題至多扣 20 分，扣至 0 分為止。

1. Let G be an arbitrary weighted, directed graph. Please describe an algorithm to detect if G existed negative cycles. And explain the time complexity of your method.
2. Explain how to find the minimum key stored in a B-Tree and how to find the predecessor of a given key stored in a B-Tree?
3. Generalize Huffman's algorithm to ternary codewords (i.e., codewords using the symbol 0,1, and 2), and prove that it yields optimal ternary codes.
4. Describe an algorithm that, given $\boldsymbol{n}$ integers in the range $\boldsymbol{0}$ to $\boldsymbol{k}$, preprocesses its input and then answers any query about how many of the n integers fall into a range[a…b] in O($\boldsymbol{1}$) time. Your algorithm should use $\theta(\boldsymbol{n}+\boldsymbol{k})$ preprocessing time.
5. Show that quicksort's best-case and mergesort's worst-case running time is $\Omega(\boldsymbol{nlgn})$.
6. We know that finding a smallest vertex cover is an NP-complete problem. Can you show that, in a given graph G=(V,E), find out a maximum subset S of E such that any two nodes in S are not adjacent in G, is a NP-Complete problem.