

長庚大學一百學年度第二學期電機所博士班演算法資格考

1. 請於答案卷第一頁依序寫下學號、姓名。
 2. 請詳細閱讀下列試題，並請標明題號依試題順序將答案書寫於答案卷上。
 3. 任何形式的作弊，本資格考以 Fail 論。
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請選擇五題作答。本次考試總分為 100 分，每錯一題至多扣 20 分，扣至 0 分為止。

1. Please describe what is LSD radix sort? And try to analysis the time complexity of LSD radix sort on a list of strings. Each character represents a key and the size of the alphabet for each key is m . The maximum length of each string is m and the number of the list is n . (20 points)
2. Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant for $n \leq 2$. Make your bounds as tight as possible, and justify your answers. (20 points)
 - A. $T(n) = 2T(n/2) + n^3$
 - B. $T(n) = T(n-1) + n$
 - C. $T(n) = 2T(n/4) + \sqrt{n}$
 - D. $T(n) = T(n-1) + \lg n$
3. We know that finding a smallest vertex cover is an NP-complete problem. Can you show that, in a given graph $G=(V,E)$, find out a maximum subset S of E such that any two nodes in S are not adjacent in G , is a NP-Complete problem. (20 points)
4. Let G be an arbitrary weighted, directed graph with a negative-weight cycle reachable from the source vertex s . Show that an infinite sequence of relaxations of the edges of G can always be constructed such that every relaxation causes a shortest-path estimate to change. (20 points)
5. Please give a recursive algorithm $\text{MATRIX-CHAIN-MULTIPLY}(A, s, i, j)$ that actually performs the optimal matrix-chain multiplication, given the sequence of matrices $\langle A_1, A_2, A_3, \dots, A_n \rangle$, the s table computed for internal use, and the indices i and j . (The initial call would be $\text{MATRIX-CHAIN-MULTIPLY}(A, s, 1, n)$.) (20 points)
6. For a sequence of operations to be performed on a data structure, the i_{th} operation costs i when i is an exact power of 2, costs 1 otherwise. Show that the cost after n operations is $O(n)$. (Amortized Analysis) (20 points)