長庚大學 102 學年度第一學期電機所博士班演算法資格考

1. 請於答案卷第一頁依序寫下學號、姓名。
2. 請詳細閱讀下列試題，並請標明題號依試題順序將答案書寫於答案卷上。
3. 任何形式的作弊，本資格考以 Fail 論。

請選擇五題作答。本次考試總分為 100 分，每錯一題至多扣 20 分，扣至 0 分為止。

1. In the GENERIC Minimum Spanning Tree(MST), let G=(V,E) be a connected, undirected graph with a real valued weight function w defined on E. Let A be a subset of E that is included in some MST for G, let (S, V-S) be any cut of G that respects A, and let (u,v) be a light edge crossing (S, V-S). Then, edge (u,v) is safe for A. Please proof the description above.
2. Please describe any algorithm that can sort n integers with O(*n*) time complexity, and also describe the constrains or limitation of the algorithm.
3. Please use the dynamic programming approach to design an algorithm to find the maximum sum in any contiguous sublist of a given list of n real values. For example, consider the list [6, 2, -14, 9, -2, 8, 4, -5]. It consists of a contiguous sublist [9, -2, 8, 4] which has the maximum sum 19. Please describe your algorithm, and show the time complexity using order notation.
4. Let $X= (x_1,x_2,\ldots,x_m)$, $Y= (y_1,y_2,\ldots,y_n)$, $Z= (z_1,z_2,\ldots,z_k)$. If Z is the Longest Common Subsequence (LCS) of X and Y. Please proof that if $x_m=y_n$, then $z_k=x_m=y_n$ and Z_{k-1} is an LCS of X_{m-1} and Y_{n-1}.
5. We know that binary search tree keep the good relationship for the order of the nodes. Please describe a sorting algorithm using a binary search tree. And show us the best/worst time complexity of your method. Is it stable?
6. Please proof that $n_0=n_2+1$ in a binary tree for n_0 is the number of leave nodes, n_1 is the number of nodes with one child and n_2 is the number of nodes with 2 children. If a complete binary tree with n nodes is represented sequentially in an array, then for any node with index i, $1 \le i \le n$. If we use the same scenario for a skewed tree of depth k, how many spaces in the array will be wasted?
7. We know that finding a smallest vertex cover is an NP-complete problem. Can you show that, in a given graph G=(V,E), find out a maximum subset S of E such that any two nodes in S are not adjacent in G, is a NP-Complete problem.